A. Georgaki and G. Kouroupetroglou (Eds.), Proceedings ICMC|SMC|2014, 14-20 September 2014, Athens, Greece

Computing Musical Meter – an Approach to an Integrated Formal Description

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ABSTRACT

Musical meter is described as an abstract temporal template for the timing of concrete musical events. The essential structural properties of meter are specified to prepare the development of an analytic method for the formalisation of meter in terms of numeric structures. An elaborated notion of the hierarchic architecture of meter supports an integrated numerical description of multiplicative and additive meters which may serve as a consistent resource for advanced computer-aided research on meter and for algorithmic composition. Likewise aiming at theoretic validity and implementability a set of underlying formal principles is suggested to guide the method of description. It is supposed as one possible axiomatic approach in order to avoid limitations on systematic flexibility and expandability as well as on artistic usability.

1. INTRODUCTION

The composition of temporal structures as an aspect of a compositional process can involve the usage of meter as an abstract temporal template or pattern to time concrete rhythms and musical figures. In this sense meter underlies the musical surface as a temporal context which influences our musical understanding: there are complex interdependencies between the sounding surface and metric interpretation which is the mapping of sound events on a cognitive temporal pattern.

Certain theoretical and psychological aspects of meter have been taken into account to support research on these interdependencies: amongst others an integrated notion of meter and tempo (see e.g. [1],[2]) and metrical cues in rhythmical patterns (see e.g. [3],[4]). The incorporation of those specific ideas by a common theoretical but implementable model could be useful for the integration in systems for algorithmic composition. As its structures and descriptions are generated algorithmically they require a thorough formal examination and a common formalisation.

In this paper a generic analytic method to describe meter in terms of hierarchical numeric structures is suggested. It integrates the description of multiplicative and additive meters which can serve as a basis for further formalisation of the mentioned aspects of rhythm and meter. The system of description is based on an upgradeable set of formal principles. It is supposed as one (but not the only) possible axiomatic approach in order to first achieve a consistent resource for further studies and secondly to avoid any kind of limitation. As there could be different opinions on the formal implications of musical meter the system might be flexible and expandable to include further advanced concepts. For composers and musicologists an open system might as well be more suitable to use in different contexts and environments.

For an easy introduction on the matter the first examples are shown in common musical notation. The following numerical examples should be regarded in a flexible relation to possible ways of notation. They show the essential formal properties as they can be implemented in advanced research or applications.

2. A GENERIC DESCRIPTION OF METER

Meter will be described here as a cyclic sequence of metrical periods. They are vertically superposed as a multiple and theoretically infinite stratification of metric levels with fixed period ratios (e.g. 2 : 1, see Figure 1) between adjacent levels. This leads to a simple, but hierarchic grouping structure [1] .

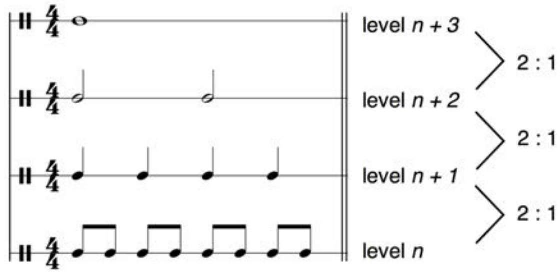

Figure 1. Metric levels and hierarchic grouping structure

If the grouping structure of a level is contradicted e.g. by a rhythmic cue on the musical surface (see Figure 2, next page), syncopation arises.

As the notion of meter described here is cyclic, it is possible to establish metrically what has been a syncopation before. Then it would be more accurate to write the grouping structure – or as it will be called here, the *layer structure* – right into the time signature (see Figure 3, next page).

[1] see e.g. [5] page 13 et seqq.

Figure 2. Rhythmic contradiction of metric grouping

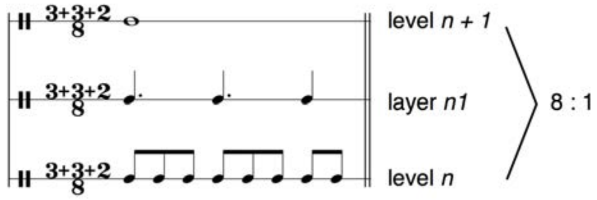

Figure 3. Layer structure and time signature

A *layer* will be defined as a non-isochronous stratum in a metric hierarchy. The durations of its metric periods are regularly changing. In the case of Figure 3 it is the first (and only) layer on level n. It can also correspond to a perceived beat if the tempo of this layer is moderate. As a consequence we now have an eight-to-one-relation between two adjacent levels. In this terminology the ratio of two levels is always fixed but there is no fixed ratio between a level and a layer. Due to its additive structure the resulting meter cannot be mirrored in time without changing it. The metric hierarchy of additive meters generally has a sandwich-like structure with alternating isochronous and non-isochronous strata (respectively levels and layers).

As a numerical formalisation *1 0 0 1 0 0 1 0* is suggested for the layer structure in Figure 3. There is one layer between the two adjacent levels. The level itself can be regarded as a zero-layer.

2.1 Assumtion: metrical groupings contain two or three elements

The assumtion that metrical groupings contain two or three elements is supported by the following experience: as soon as there are four beats or pulses [2] they tend to group into two groups of two, or a group with five elements tends to fall apart into two and three or vice versa [3]. This is also related to the phenomenon called *subjective grouping* or *subjective rhythmisation* which is already well researched in music psychology [4]. If the resulting groups have equal sizes like a subdivided group of four elements the grouping and the resulting meter will be called *multiplicative* (2 x 2). Cases like a subdivided group of five elements where the resulting groups have different sizes will be called *additive* (e.g. 2 + 3 or 3 + 2).

The more or less complex alternation of stronger and weaker metrical periods as a result of a metric hierarchy is a common property of meter. Nevertheless there may be musical meters refusing this dynamics and may contradict the assumption like a five-period-meter with a cycle of a strong period and four equal weak periods. This should be kept in mind but they will not be discussed in this paper.

2.2 Multiplicative meters

The possibilities of pure multiplicative meters up to 24 periods per full metric cycle are listed in Table 1. The numeric notation shows the relations of period durations of adjacent levels. For example the only possible meter with four pulses has three levels with two periods of level 0 (the pulse-level) combined in level 1 and two periods of level 1 combined in level 2. It has to be made clear in which order the multiplicators relate to the indices of the levels. Here the order is from lower/faster levels to higher/slower levels. Hence, in the first example of the two six-period-meters level 1 contains three periods of level 0 and level 2 combines two periods of level 1.

periods	multiplicative meters	time signature example
4	2 x 2	$\frac{2}{4}$ ($\frac{1}{8}$ pulse)
6	3 x 2 2 x 3	$\frac{6}{8}$ ($\frac{1}{8}$ pulse) $\frac{3}{4}$ ($\frac{1}{8}$ pulse)
8	2 x 2 x 2	$\frac{4}{4}$ ($\frac{1}{8}$ pulse)
9	3 x 3	$\frac{9}{8}$ ($\frac{1}{8}$ pulse)
12	3 x 2 x 2 2 x 3 x 2 2 x 2 x 3	$\frac{12}{8}$ ($\frac{1}{8}$ pulse) $\frac{6}{8}$ ($\frac{1}{16}$ pulse) $\frac{3}{4}$ ($\frac{1}{16}$ pulse)
16	2 x 2 x 2 x 2	$\frac{4}{4}$ ($\frac{1}{16}$ pulse)
18	3 x 3 x 2 3 x 2 x 3 2 x 3 x 3	$\frac{6}{4}$ ($\frac{1}{8}$ triplet pulse) $\frac{3}{2}$ ($\frac{1}{8}$ triplet pulse) $\frac{9}{4}$ ($\frac{1}{8}$ pulse)
24	3 x 2 x 2 x 2 2 x 3 x 2 x 2 2 x 2 x 3 x 2 2 x 2 x 2 x 3	$\frac{4}{4}$ ($\frac{1}{16}$ triplet pulse) $\frac{12}{8}$ ($\frac{1}{16}$ pulse) $\frac{6}{4}$ ($\frac{1}{16}$ pulse) $\frac{3}{2}$ ($\frac{1}{16}$ pulse)

Table 1. Multiplicative meters up to 24 periods per metric cycle

The examples of time signatures just give a hint to relate

[2] In this paper a pulse is defined as the shortest or fastest metric period. The pulse-level is defined as the fastest or lowest level in a metric hierarchy (level 0) and is always isochronous. A beat can be subdivided into faster or shorter metric periods. The beat-level or beat-layer is the most salient metric level or layer and could be isochronous or non-isochronous.

[3] This is similar to simple versus compound subdivision, but not the same because the different subdivisions form groups of the same size ($\frac{2}{2}$ vs. $\frac{3}{3}$) while five elements divide into two groups of different sizes ($\frac{2}{1}$ vs. $\frac{3}{1}$)

[4] see e.g. [2] page 418 et seqq.

a certain meter to a possible musical notation. Taking other musical context into account there could be different solutions. A meter with only four pulses in a full cycle could also be notated in $\frac{2}{8}$ time signature and the pulse duration would then be $\frac{1}{16}$.

2.3 Additive meters – an exploration of possible layer structures

The number of periods in a layer structure is always corresponding to the ratio between two adjacent metric levels. In Table 2 all possible additive groupings, their permutations and the resulting layer structures are listed for $n < 10$ periods fitting in one *superlevel*[5] period.

periods	possible additive groupings by the first layer	possible layer structures
5	2 3 3 2	1 0 1 0 0 1 0 0 1 0
7	2 2 3 2 3 2 3 2 2	1 0 1 0 1 0 0 1 0 1 0 0 1 0 1 0 0 1 0 1 0
8	2 3 3 3 2 3 3 3 2	1 0 1 0 0 1 0 0 1 0 0 1 0 1 0 0 1 0 0 1 0 0 1 0
9	2 2 2 3 2 2 3 2 2 3 2 2 3 2 2 2	2 0 1 0 2 0 1 0 0 2 0 1 0 2 0 0 1 0 2 0 1 0 0 2 0 1 0 2 0 0 1 0 2 0 1 0

Table 2.

Five is the smallest number of metric periods to divide into additive groups of two or three. Six periods are omitted because the possible groupings are (like with four periods) all multiplicative. A level with nine periods per superlevel period is the first case with more than one layer: according to the assumption the groups formed by the first layer become elements of a second order grouping which can be regarded as a second layer. This layer can contain metrical groupings with more complex ratios: in the case of nine periods per superlevel period $4 : 5$ and $5 : 4$ ratios will occur among period durations of the second layer:

$\underline{2\ 2}\ \underline{2\ 3}$	$\underline{2\ 2}\ \underline{3\ 2}$	$\underline{2\ 3}\ \underline{2\ 2}$	$\underline{3\ 2}\ \underline{2\ 2}$	layer 1
| |	| |	| |	| |	
4 5	4 5	5 4	5 4	layer 2

In additive groupings of ten periods yet another interference is introduced. There are cases of second order groups containing additive first order groupings but having equal sizes:

$\underline{2\ 3}\ \underline{2\ 3}$	$\underline{2\ 3}\ \underline{3\ 2}$	layer 1
| |	| |	
5 5	5 5	a level?

[5] the level $n + 1$ will be called the *superlevel* of level n

According to the definitions these groups are not formed by a layer. Elements with the same size or duration form an isochronous stratum which has to be distinguished from a layer shaping non-isochronous groups by definition.

Nevertheless this leads to two different types of symmetry. The grouping 2 + 3 + 2 + 3 holds a translational symmetry between the second order groups while in the case of 2 + 3 + 3 + 2 these groups mirror each other. The former can be broken down to two periods of a level each with five periods and a layer structure *1 0 1 0 0*, hence it is not a case of a valid layer structure with ten periods. If we assume the same for the latter, the layer structure would change from the first to the second period of the five-period-level (from *1 0 1 0 0* to *1 0 0 1 0*).

In terms of musical notation this could be a change of time signature e.g. from $\frac{2+3}{8}$ to $\frac{3+2}{8}$. As soon as this metrical structure would be repeated a couple of times it would make more sense to combine the structure into one time signature $\frac{2+3}{8} + \frac{3+2}{8}$ than to notate an alternating time signature every bar or measure. This would better reflect the structure of the full metrical cycle. The five-period-level would then be the half-measure level.

Generally a *measure* in this context can be defined as a period of a level n containing a metric structure with at least one lower level which is exactly repeated in the next period of level n. In other words metrical groupings within different periods of a measure are always translational symmetric to each other. The mentioned half-measure level can be regarded as a case of an *intermediate level* whose successive periods contain different, alternating metric structures. Additive meters with mirror symmetries within a measure period like in Figure 7 b) are of certain speciality as they have a property in common with multiplicative meters: they do not change when they are mirrored in time i.e. played backwards. They are not the only cases where intermediate levels occur, as we will see later.

periods	10
possible additive groupings by the first layer	2 2 3 3 2 3 3 2 3 2 2 3 3 3 2 2
possible layer structures	2 0 1 0 2 0 0 1 0 0 2 0 0 1 0 0 2 0 1 0
possible layer structures with an intermediate level	2 0 1 0 0 2 0 0 1 0 2 0 0 1 0 2 0 1 0 0

Table 3.

With ten periods per superlevel period both of the two possible structures with an intermediate level contain mirror symmetries (Table 3). In terms of their inherent metrical structure the periods of the intermediate levels are alternating mirrors of each other.

From eleven periods upwards alternative second order groupings lead to a steeper exponential growth of the number of possible layer structures. With eleven periods there are nine alternative additive groupings by the first layer but already 14 possible layer structures with second order groupings upon the first layer (Table 4). Layer structures with intermediate levels are not possible because eleven is a prime number.

periods	11
possible additive groupings by the first layer	2 2 2 2 3 2 2 2 3 2 2 2 3 2 2 2 3 2 2 2 3 2 2 2 2 2 3 3 3 3 2 3 3 3 3 2 3 3 3 3 2
possible layer structures	2 0 1 0 2 0 1 0 1 0 0 2 0 1 0 1 0 2 0 1 0 0 2 0 1 0 2 0 1 0 0 1 0 2 0 1 0 1 0 2 0 0 1 0 2 0 1 0 2 0 0 1 0 1 0 2 0 1 0 1 0 0 2 0 1 0 2 0 1 0 0 2 0 1 0 1 0 2 0 1 0 0 1 0 2 0 1 0 2 0 0 1 0 2 0 1 0 1 0 2 0 0 1 0 1 0 2 0 1 0 2 0 1 0 0 2 0 0 1 0 0 2 0 0 1 0 2 0 0 1 0 0 2 0 0 1 0 0 2 0 1 0 0 2 0 0 1 0 0 2 0 0 1 0

Table 4.

The possible additive substructures of a level with twelve periods show two different examples of intermediate levels. In these cases there are no mirror symmetries but characteristic changes of the period durations of the first layer (Table 5). The number of first-layer-periods alternates in both cases between two and three every period of the intermediate level. The resultung meters could be notated as $\frac{3}{4}+\frac{6}{8}$ and $\frac{6}{8}+\frac{3}{4}$.

periods	12
possible layer structures	2 0 1 0 2 0 1 0 0 1 0 0 2 0 1 0 2 0 0 1 0 1 0 0 2 0 1 0 1 0 0 2 0 1 0 0 2 0 1 0 2 0 0 1 0 0 1 0 2 0 1 0 1 0 0 2 0 0 1 0 2 0 1 0 0 2 0 1 0 1 0 0 2 0 1 0 0 1 0 2 0 1 0 0 2 0 1 0 0 2 0 1 0 0 1 0 2 0 1 0 0 1 0 2 0 0 1 0 2 0 1 0 0 2 0 0 1 0 1 0 2 0 1 0 0 1 0 0 2 0 1 0 2 0 0 1 0 2 0 1 0 1 0 0 2 0 0 1 0 1 0 2 0 1 0 0 2 0 0 1 0 2 0 1 0 0 1 0 2 0 0 1 0 1 0 2 0 0 1 0 2 0 0 1 0 2 0 0 1 0 1 0 2 0 0 1 0 1 0 0 2 0 1 0 2 0 0 1 0 0 1 0 2 0 1 0
possible layer structures with an intermediate level	2 0 1 0 1 0 2 0 0 1 0 0 2 0 0 1 0 0 2 0 1 0 1 0

Table 5.

Generally intermediate levels divide and shape oscillating metric substructures which are combined in a period of a superlevel. A look e.g. at the six possible layer structures of a 15-period-meter containing an intermediate level in Table 6 reveals oscillations between the two different five-period-structures (see Table 2). There are always three periods of an intermediate level containing the possible combinations with three elements of two types.

periods	possible layer structures with an intermediate level	combinations of the two possible five-period layer structures
15	2 0 1 0 0 2 0 1 0 0 2 0 0 1 0 2 0 1 0 0 2 0 0 1 0 2 0 1 0 0 2 0 1 0 0 2 0 0 1 0 2 0 0 1 0 2 0 0 1 0 2 0 1 0 0 2 0 1 0 0 2 0 0 1 0 2 0 1 0 0 2 0 0 1 0 2 0 0 1 0 2 0 0 1 0 2 0 1 0 0	a a b a b a a b b b a a b a b b b a

Table 6.

To illustrate the combinatory explosion of possibilities with a growing number of periods, the numbers of alternative layer structures in the area between 13 and 24 periods per superlevel are listed in Table 7 (see next page). All metric cycles with a non-prime number of periods in this range contain layer structures with intermediate levels. Their numbers and correspondent examples are shown in brackets. They form a subset of all possible layer structures the numbers of which are listed first. The rightmost column gives information about possible depths of metric stratification in these additive structures. There could be second and third order groupings according to the assumption in 2.1.

Additive meters with a correspondent quantity of periods in a full metric cycle are rare and may be found only among indian talas or in certain bulgarian meters [6]. However, a combinatoric exploration can be regarded as a prerequisite for subsequent filtering: an algorithmic selection process according to a certain compositional aesthetics.

2.4 Hybrid meters

Meters made up from combined multiplicative and additive structures are suggested to be called *hybrid meters*. The possible examples of these kind of meters with cycles containing up to 24 periods are listed in Table 8 (see next page). The second column shows the multiplicative metric structures. As in Table 1 the numeric notation specifies the relations of period durations of adjacent levels. The additive components can be specified with the possible layer structures of certain levels. For example, the four possible hybrid meters with ten periods can be described as follows. In the first two variants level 0 contains five pulses which can be grouped in two possible ways (see structural variants in the third column, see also Table 2). As a result there is one layer between level 0 and level 1. In the two other variants level 2 combines five periods of level 1. The two possible ways of grouping constitute a layer between level 1 and level 2.

periods	possible layer structures (with intermediate levels)	examples	number of layers
13	32	2001020100100	2
14	34 (6)	20010201020100 (20101002010010)	2
15	57 (6)	200102010200100 (201002001020100)	2
16	87 (6)	2010100200100100 (2010010020010100)	2
17	149	30102010301020010	2 or 3
18	201 (18)	201010201002010010 (301020010301020100)	2 or 3
19	332	2010102001001020100	2 or 3
20	457 (54)	30102010030100102010 (30102001003010102010)	2 or 3
21	709 (24)	301020100301002010100 (201010020010102010010)	2 or 3
22	1046 (172)	2010102010010200100100 (3001002010030102010100)	2 or 3
23	1776	30101002001030010020010	2 or 3
24	2775 (508)	301002010010030100201010 (200101002001001020100100)	2 or 3

Table 7. Combinatory explosion of grouping possibilities between 13 and 24 periods per metric cycle

3. SUMMARY AND CONCLUSIONS

Meter implies a hierarchically structured temporal context for rhythmic interpretation. Some formal implications of metrical grouping are defined here as a line-up of principles. The resulting sets of possible metric structures are generated algorithmically. The discussion and analysis of these sets constitute different types of meters. Multiplicative and additive meters can be described with the same generic parameters. Another class containing both multiplicative and additive groupings is characterised as hybrid meters. Additionally some specific properties of certain meters like different types of symmetries are revealed and analysed.

Possible applications can be conceived in the field of algorithmic composition and in other disciplines dealing with structured time. The generation of complete sets of metric structures with predefined properties can provide a source for algorithmic selection processes or to explore possible metric interpretations of a series of inter-onset-intervals.

periods	hybrid meters	structural variants	number of hybrid meters
10	5 x 2 2 x 5	2 2	4
14	7 x 2 2 x 6	3 3	6
15	5 x 3 3 x 5	2 2	4
16	8 x 2 2 x 8	3 3	6
18	9 x 2 2 x 9	4 4	8
20	10 x 2 2 x 10 5 x 2 x 2 2 x 5 x 2 2 x 2 x 5	4 4 2 2 2	14
21	7 x 3 3 x 7	3 3	6
22	11 x 2 2 x 11	14 14	28
24	12 x 2 8 x 3 3 x 8 2 x 12	20 3 3 20	46

Table 8. Hybrid meters up to 24 period per metric cycle

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